

PENETRABILITY AND LIFETIME PREDICTIONS  
FOR SPONTANEOUS EMISSION OF HEAVY IONS  
FROM ATOMIC NUCLEI

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Up to now only penetrabilities, which are not measurable quantities, have been used by experimentalists to choose the most likely spontaneously emitted clusters. On the example of  $^{222}\text{Ra}$ ,  $^{232}\text{Th}$ ,  $^{238,238}\text{U}$ ,  $^{244}\text{Pu}$ ,  $^{248}\text{Cm}$ ,  $^{252}\text{Cf}$ , and  $^{254}\text{Fm}$  parent nuclei it is shown that sometimes this method can lead to wrong predictions in comparison with the lifetime calculations, in the framework of the model developed by the authors.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Предсказание проницаемости и времени жизни  
для спонтанной эмиссии тяжелых ионов  
из атомных ядер

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До настоящего времени экспериментаторы использовали только расчеты проницаемости /которые представляют собой неизмеряемые величины/ для выбора самых вероятностных кластеров, спонтанно испускаемых ядрами. На примере родительских ядер  $^{222}\text{Ra}$ ,  $^{232}\text{Th}$ ,  $^{238,238}\text{U}$ ,  $^{244}\text{Pu}$ ,  $^{248}\text{Cm}$ ,  $^{252}\text{Cf}$  и  $^{254}\text{Fm}$  показано, что иногда этот метод может приводить к неправильным предположениям по сравнению с расчетом времени жизни в рамках развиваемой авторами модели.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

At the beginning of this year, Rose and Jones<sup>1/</sup> from Oxford University discovered the  $^{14}\text{C}$  radioactivity of  $^{223}\text{Ra}$ . In a run of 189 days they have observed 11 events, leading to the branching ratio relative to the  $\alpha$  decay

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of  $(8.5 \pm 2.5) \cdot 10^{-10}$  and to  $T \sim 10^{15}$  s half-life against this process for  $^{223}\text{Ra}$ . There are two confirmations of this experiment, coming from Moscow<sup>/2/</sup> and Orsay<sup>/3/</sup>, in which the time of measurement was reduced to  $\sim 30$  and 5 days, respectively.

This is the first evidence for one of many kinds of new decay modes predicted since 1980<sup>/4,5/</sup>.  $^{14}\text{C}$  emission from  $^{222}\text{Ra}$  and  $^{224}\text{Ra}$  parent-nuclei (which are present among other examples in Figure 7 of ref.<sup>/4/</sup>) has been reported recently by Price et al.<sup>/6/</sup> The shell effects in the region of the double magic  $^{208}\text{Pb}$  daughter are responsible for these phenomena occurring with maximum intensity in trans-lead parent nuclei. Nevertheless it is also met in the trans-tin region<sup>/7/</sup>, where the next double magic daughter  $^{132}\text{Sn}$  plays the major role. Almost all nuclei are metastable with respect to several decay modes, but if the lifetime is longer than  $10^{30}$  s, one can say that from the practical point of view, the corresponding nucleus is stable.

For a very asymmetric splitting



where the daughter has the subscript 1 and the emitted cluster 2, we have made predictions<sup>/4/</sup> across the nuclear table on the basis of penetrability calculations. In the same time, a closed relationship for the lifetime was found<sup>/5/</sup> by using the assumption that the emission process is a superasymmetric fission phenomenon<sup>/8,9/</sup>. It was possible to derive this formula after a successful application of the numerical methods used in fission theory<sup>/10,11/</sup> to the  $\alpha$  decay<sup>/8/</sup>. In 1983 it was improved to account for angular momentum and small excitation energy effects and was used to predict  $^5\text{He}$  and  $\beta$ -delayed  $^5\text{He}$  radioactivity<sup>/12/</sup> with a zero point vibration energy,  $E_v = 0.51$  MeV, obtained from a fit with experimental data on 376  $\alpha$ -emitters.

When the branching ratio for the  $^{14}\text{C}$  radioactivity of  $^{223}\text{Ra}$  was available<sup>/1/</sup>, the relationship  $E_v = 0.1275 A_2$  could also account<sup>/9/</sup> for the new experimental point<sup>/1/</sup>. Recently<sup>/7,13/</sup> we found that a better fit could be obtained for 380  $\alpha$ -emitters and  $^{14}\text{C}$  radioactivity, if we use the following equation:

$$E_v = Q \left[ 0.056 + 0.039 \exp\left(\frac{A_2 - 4}{2.5}\right) \right]; A_2 > 4, Q > 0, \quad (2)$$

where  $Q$  is the released energy computed from the new experimental mass tables<sup>/14/</sup>.

The purpose of this paper is to compare the penetrability with the lifetime calculations.

The half-life of a metastable system is given by

$$T = \hbar \ln 2 / \Gamma, \quad (3)$$

where  $\Gamma$  is the disintegration partial width. In the relationship

$$\Gamma = 2\gamma^2 \cdot P' \quad (4)$$

used in the R-matrix theory of  $\alpha$ -decay, the reduced width  $\gamma^2$  is proportional to the cluster preformation probability. For  $\alpha$  particle one can compute this quantity by using microscopically determined wave functions, but for heavier ions the similar procedure is not developed up to now. Due to this fact, only the penetrability  $P'$  was computed in refs.<sup>/1-4/</sup> by assuming that the emitted cluster is reduced to a point going by quantum-mechanical tunneling through the potential barrier which is mainly due to the Coulomb repulsion. By using the semiclassical WKB theory, one has:

$$P' = \exp \left\{ - \frac{2}{\hbar} \int_{R_t}^{R_{out}} \sqrt{2\mu [V(R) - Q]} dR \right\}, \quad (5)$$

where  $\mu$  is the reduced mass  $\mu = \frac{A_1 A_2}{A} m$ ;  $m$  is the nucleon mass;  $V(R)$  is the potential barrier;  $R_t$  is of the order of the touching point distance:

$$R_t \approx R_1 + R_2; \quad R_j = r_0 A_j^{1/3} \quad (j = 1, 2) \quad (6)$$

and  $R_{out}$  is found from the equation  $V(R_{out}) = 0$ . The distance  $R_t$  depends on the adopted value of the radius constant  $r_0$ . Hence Rose and Jones<sup>/1/</sup> have obtained 3 different  $P'$  values for  $r_0 = 1.15; 1.20$  and  $1.25$  fm. At a distance  $R \geq R_t$  the contribution of the strong interaction is small in comparison with that of the Coulomb field. In this region one has only the tail of the optical potential determined from scattering experiments.

In fission theory<sup>/11/</sup> the disintegration width is given by

$$\Gamma = \hbar \nu \cdot P = \frac{E_v}{\hbar} P, \quad (7)$$

where  $\nu = \frac{\omega}{2\pi} = \frac{2E_v}{h}$  represents the number of assaults on the barrier per second (the characteristic frequency of

the collective mode) and  $E_v = \frac{\hbar\omega}{2}$  is the zero point vibration energy. This is a phenomenological quantity obtained by fitting the experimental half-lives data.

Now the WKB penetrability

$$P = \exp(-K) \quad (8)$$

can be expressed as a product of two terms due to the fact that in the action integral

$$K = K_{ov} + K_s = \frac{2}{\hbar} \left( \int_{R_a}^{R_t} dr + \int_{R_t}^{R_b} dr \right) \sqrt{2\mu[E(r) - Q]} \quad (9)$$

one has not only the term  $K_s$  corresponding to separated fragments but also the contribution of the overlapping region  $K_{ov}$  which is not present in  $P'$ . In this equation one has

$$Q' = Q + E_v; \quad E(R_a) = E(R_b) = Q'. \quad (10)$$

The energy barrier  $E(r)$  can be computed by the Strutinsky macroscopic-microscopic method<sup>/10/</sup> adapted for the superasymmetric splitting<sup>/8/</sup> by using model parameters obtained from a fit with experimental data on masses, fusion and fission barriers, electron scattering experiments, etc. The emitted cluster is no longer a point in a potential, it has a spatial extension with a radius  $R_2$  and  $E(r)$  is the deformation energy of the two bodies. For  $\alpha$  decay  $K_{ov}$  is only a small percent of  $K_s$ , but it increases with increasing mass number  $A_2$ . Consequently, the zero point vibration frequency  $E_v$  from eq.(7) does not play the role of the preformation probability  $\gamma^2$  of eq.(4).

For a set of experimental data on 376  $\alpha$ -emitters<sup>/15/</sup> we have found that the variation of  $E_v$  with the neutron number  $N_1$  of the daughter nucleus is much smaller than that of  $\gamma^2$ . This "stability" of the parameter  $E_v$  is a useful property when the method is employed to predict the lifetime for a new decay mode. It is the consequence of the fact that in eq. (7), when  $E_v$  is increased,  $P$  increases exponentially through eqs.(8), (9), and (10).

The comparison between the two methods employed up to now is illustrated in the Figure for the spontaneous emission of various even-even heavy ions from some even-even parent nuclei.

The decimal logarithm of the penetrability spectra is taken from Fig.7 of the Ref.<sup>/4/</sup> where the half of

the natural logarithm ( $\frac{1}{2} \ln P'$ ) has been plotted. The lifetime spectra are calculated with the analytical relationship given in Ref. <sup>5,9,12/</sup> and eq.(2). The vertical scales of  $\log P'$  and  $\log T(s)$  curves are chosen in such a way that the experimental  $\alpha$ -decay lifetime  $T$  is at the same level with the corresponding penetrability  $P'_{\text{cor}}$ , corrected for  $2\gamma_a^2 \neq 1$  MeV:  $\log T^{\text{exp}} = -21.34 - \log P - \log(2\gamma_a^2)$ .

From the measured branching ratio <sup>1-3/</sup> for the  $^{14}\text{C}$  radioactivity of  $^{223}\text{Ra}$  relative to the  $\alpha$  decay:

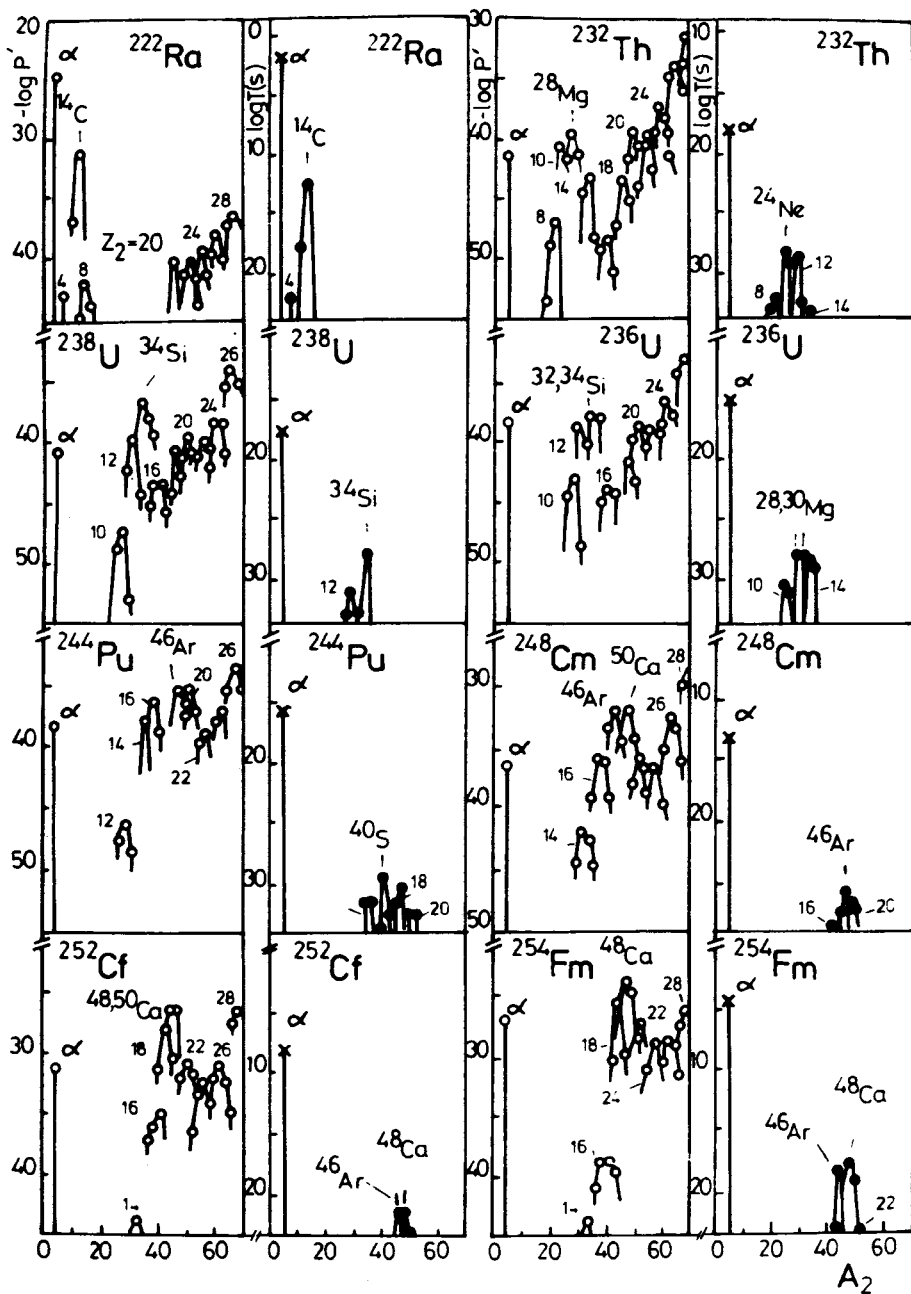
$$\frac{\Gamma}{\Gamma_a} = \frac{T_a}{T} = \frac{\gamma^2}{\gamma_a^2} \cdot \frac{P'}{P'_a} = \frac{E_v}{E_{va}} \cdot \frac{P}{P_a} \quad (11)$$

which was of the order of  $10^{-9}$  and the computed penetrability ratio  $P'/P'_a \approx 10^{-8}$ , one has the preformation probability ratio  $\gamma^2/\gamma_a^2 \approx 10^{-6}$  and it is expected that this ratio will be much smaller for emitted ions heavier than  $^{14}\text{C}$ . But if we use a more realistic interaction barrier including also the overlapping region of two fragments, one obtains the larger ratio ( $\gamma^2/\gamma_a^2$ ).

By using Coulomb plus proximity potential barriers, a value of the ratio  $P/P_a$ , very close to the experimental branching ratios for  $^{14}\text{C}$  emission from  $^{222,223,224}\text{Ra}$  (which means that  $E_v = (1.4 \div 11.3) E_v$  in this model) has been reported <sup>16/</sup>. In our case the corresponding zero point vibration energy ratio is of the order  $E_v/E_{va} \approx 3.5$  because we use very high barrier heights. In the framework of  $Y+EM$  plus shell corrections <sup>8,5,13/</sup> we obtain  $E_v/E_{va} \approx 1/12$  comparable with  $1/11.3$  <sup>16/</sup>. This has to be compared with  $\gamma^2/\gamma_a^2 \approx 10^{-6}$  <sup>1/</sup>, in order to see the stability of the parameter  $E_v$ .

The argument used by some experimentalists <sup>1-3/</sup> that they found the maximum of the ratio  $P'/P'_a$ , does not imply necessarily that the branching ratio  $\Gamma/\Gamma_a$  will also have a maximum value. From the cases shown in the Figure one can see that sometimes the penetrability spectra indicate the same cluster emitted with maximum probability as the lifetime spectra (example:  $^{14}\text{C}$  from  $^{222}\text{Ra}$ ,  $^{34}\text{Si}$  from  $^{238}\text{U}$  and  $^{48}\text{Ca}$  from  $^{254}\text{Fm}$ ). But there are many cases in which from the two spectra one gets different results, like for example from  $^{244}\text{Pu}$ , instead of  $^{46}\text{Ar}$  and  $^{50}\text{Ca}$  of penetrability spectra one has  $^{40}\text{S}$  in the lifetime spectra.

In the same time there is a trend of increasing of the value of  $\log P'$  when  $A_2$  increases which is not present in the lifetime spectra. Moreover, frequently one can meet the situation in which the penetrability  $P'$  for a given heavy ion emission is even larger than that for  $\alpha$  decay, from which by no means one can say that the branching ratio  $\Gamma/\Gamma_a$  should be larger than unity.



Penetrability  $P'$ , and half-life  $T$ , spectra for the emission of even-even heavy ions from some even-even nuclei.

In conclusion, the penetrabilities do not hinder the spontaneous emission of heavy clusters and in a small region of emitted nuclei they could be used to get a rough information about the new decay modes. But in order to estimate the branching ratio directly measured in the experiment, one can use the lifetime spectra computed with our analytical relationship. The good selectivity of the lifetime spectra is well illustrated for all parent nuclei studied in this work.

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